DOCUMENT RESUME

ED 197 990 SE 034 165

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TITLE	Fluid Dynamics Applied to Streams. Physical Processes in Terrestrial and Aquatic Ecosystems, Transport Processes.
INSTITUTION	Washington Univ., Seattle. Center for Quantitative Science in Forestry, Fisheries and Wildlife.
SPONS AGENCY	National Science Foundation, Washington, D.C.
PUB DATE	Jul 77
GRANT	NSF-GZ-2980: NSF-SED74-17696
NOTE	44p.: For related documents, see SE 034 160-167 and SE 033 581-597. Contains light and broken type.
EDRS PRICE	MF01/PC02 Plus Postage.
DESCRIPTORS	Biological Sciences: Biophysics: *College Science: Ecology: Environmental Education: Higher Education: *Interdisciplinary Approach: *Learning Modules: Mathematical Applications: Physics: Science Education: *Water Resources
IDENTIFIERS	*Fluid Dynamics: *Streams

ABSTRACT

This module is part of a series designed to be used by life science students for instruction in the application of physical theory to ecosystem operation. Most modules contain computer programs which are built around a particular application of a physical process. This module deals specifically with concepts that are basic to fluid flow and streams. The objective is to be able to describe the types of flow, as well as the velocity and discharge of particular streams. These characteristics are used in comparisons of streams, stream classification, flood control, control of sedimentation, and gauging of available water for downstream use. A method used for determining discharge and velocity for a section of a stream, the concept of continuity, is presented. This concept also forms the basis for understanding the relationships of aquatic organisms to running water. Basic concepts included in this module are developed and used in later modules. (Author/CS)

PHYSICAL PROCESSES IN TERRESTRIAL AND AQUATIC ECOSYSTEMS

TRANSPORT PROCESSES

FLUID DYNAMICS APPLIED TO STREAMS

by

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FORESTRY, FISHERIES AND WILDLIFE

University of Washington

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This instructional module is part of a series on Physical Processes in Terrestrial and Aquatic Ecosystems supported by National Science Foundation Training Grant No. GZ-2980.

July 1977

PREFACE

This module was written for upper division and graduate students in the environmental sciences who are interested in the properties of fluids. Although partial differential equations are used, the approach is basically intuitive. The problems require only algebra and focus on descriptions of fluid flow and uses of conservation of mass.

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FLUID DYNAMICS APPLIED TO STREAMS

INTRODUCTION

In an earlier module of this series, some basic concepts of fluid statics were developed. It was noted that many of these concepts have an unusually strong bearing on the distribution and characteristics of aquatic organisms. A much more common circumstance, however, is one for which the fluid is in a constant state of motion. The large-scale motion of air through vegetation and the flow of rivers are notable examples. Perhaps of even greater importance is small-scale fluid motion such as whirlpools or eddies commonly observed in flowing streams. For example, extremely turbulent flow (*see* the section "Laminar and Turbulent Flow") precludes the development of larger filamentous algae and organisms with feathery appendages. Small eddies may lead to deposition of organic material and the development of communities based on this food source. Description of the flow is also important for determining the forces (due to the flow of the water) acting on organisms in the water and how these may effect their characteristics and distribution. Finally, the description of transport of dissolved and undissolved materials is dependent on description of the associated flow.

The study of fluids under all conditions of motion is known as fluid dynamics. (Fluid statics and fluid dynamics together constitute the discipline known as fluid mechanics.) The scope of fluid dynamics is tremendous, and, in general, describing a particular flow situation "under all conditions of motion" is either not possible or impractical. (Certainly it is beyond the scope of an introductory treatment.) In an attempt to understand why this is so, consider some common flow phenomena:

the plume from a smokestack, the evolution of a cumulus cloud, the flow of a river, or the formation of breakers along an ocean beach. All of these flows have in common a very irregular or random nature. Now, it is possible to formulate a set of equations which in general describe the motion of any small fluid parcel. However, due to the nature of the flow just described, it is not feasible to apply such equations to all points in time and space. While introduction of statistical techniques makes the problem tractable in some cases, their application and the resultant solutions remain beyond the scope of this module.

It becomes advisable, then, for the ecologist to simplify the physical description of the model in order to obtain a set of equations (i.e., a mathematical model) which is more tractable. The usual approach employs two concepts. The first is to average out the effects of the random fluctuations in the flow by simple decomposition of all quantities into mean values and fluctuations with zero means (*see* the section "Steady and Unsteady Flow"). The second involves choosing situations in which enough terms of the resulting equations are made insignificant to allow more simple, and hence revealing, solutions (*see* the four sections starting with "Uniform and Nonuniform Flow"). Of course, extreme care must be taken in this reduction to avoid removal of essential characteristics. Although this process requires a great deal of practice, skillful simplification rewards the researcher with results that are valid under a variety of conditions.

The equations alluded to previously as describing fluid flow are based on two conservation laws: those for mass and momentum. The momentum equations, in their most fundamental form, equate the rate of change of momentum of a fluid parcel to the forces acting on the fluid. Conservation of mass, of course, is based on the principle that matter cannot be created or destroyed by ordinary means. The simple application of the concept of conservation of mass (which leads to the continuity equation) yields both a general descriptive understanding of a particular flow and some useful applications in hydrology and limnology. Consequently, the latter part of this module focuses exclusively on continuity and its applications. A simple one-dimensional description is followed by derivation in three-dimensional space and a discussion of the continuity equation in other coordinate systems. A more complete discussion of the entire set of equations for fluid motion is deferred for the present time.

FLOW DESCRIPTIONS

Imagine yourself in a small stream with a relatively smooth streambed and irregular bank with large stones and eddy pools. This stream is in some places both broad and uniform in width; in others, a narrowing restriction occurs between two hard rocks. You are now faced with the problem of classifying and understanding flow patterns over a long reach of the stream.

For example, you may be interested in examining the possible physical flow effects on a community of insect larvae such as stoneflies or mayflies or on a community of attached algae. Or you may be interested in the influence of flow on where fish lay their eggs or even how the flow may determine the survival of the eggs. As you read through the flow descriptions think of these examples or others with which you are familiar and try to relate them to the concept being explored.

Laminar and Turbulent Flow

Fluid flows of interest in both biological and engineering applications are generally described as turbulent. For our purposes, "turbulent motion may be visualized as a system of eddies (vortices) of varying scale (size) and intensity (rotational velocity) superimposed upon the mean flow," (Eagleson 1970). These macroscopic fluid parcels move about at random, transporting their cargoes of heat, mass, and momentum. This randomness complicates mathematical solution of the flow problem. It enhances, however, the transport properties of the fluid as compared to the other flow regime of interest: laminar flow. Laminar flow can be distinguished by its dependence upon molecular transport processes, or conduction, for transport phenomena. Thus, on a macroscopic level, laminar flow appears very predictable and orderly. The accompanying diagram illustrates the contrast. If one were to place a series of markers equidistant along a line perpendicular to the mean flow and trace their paths, one would record trajectories similar to those illustrated for the two types of flow.

Streamlines

Laminar Turbulent

The lines indicate the paths of particles.

Let us return to our stream of the previous section. In the center, the water appears to flow along in a relatively orderly or streamlined manner. Near the rocky edges, irregularities in the form of riffles and eddying motion are evident. While one's first inclination might be to label the flow in these areas as laminar and turbulent, respectively, both flows are probably turbulent (*see* the next section). In actual fact, the difference in these flows is due to differences in the intensity and scale of the turbulence mentioned earlier. In the center of the stream, while the scale of the turbulence may be large compared to molecular dimensions, it can still be too small to be observed by the unaided eye. As a practical matter, time averaged quantities of interest such as velocity and temperature are used (*see* the section "Steady and Unsteady Flow"), which allow the more straightforward analysis applicable to laminar flow to be applied. Care must be exercised in using this approach, however, when the organisms involved are sensitive not only to the mean quantities, but also to their instantaneous values (e.g., very small algae will be affected by the smallest eddies).

Reynolds Number

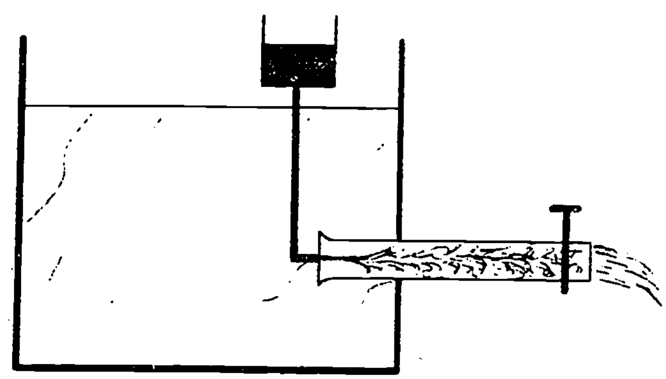
Laminar flow can become turbulent, and vice versa, as the conditions of the flow change. The onset of this transition depends on a combination of many factors, including the viscosity of the fluid, velocity of flow, channel configuration and roughness, and channel size. The Reynolds number, Re, which includes the effects of viscosity, velocity, dimension and density, is commonly used to determine whether a fluid flow can be considered laminar or must be treated as turbulent.

In 1883 Osborne Reynolds determined that, in order for two geometrically similar flows to be dynamically similar, a certain dimensionless number must be the same in both cases. This number is given by the expression:

$$\frac{v\ell\rho}{\mu} = Re \qquad (1)$$

where v is velocity, ℓ is a linear dimension, ρ is mass density and μ is the viscosity. Re is commonly called the Reynolds number and is the ratio of the momentum to the frictional forces of the flow. To determine the importance of this number for describing flow, Reynolds conducted a series of experiments on the flow of water through glass tubes. As shown in the following figure, a glass tube was mounted with the bell-mouthed end in a tank and the other end outside with a valve to control the flow. A dye jet was arranged near the smooth bell-mouthed entrance so that a fine stream of dye could be injected. The average velocity, v, and the diameter, D, were used to determine the Reynolds number, $Re = vD\rho/\mu$. For low velocity flows the dye stream moved in a straight line, indicating that the flow was laminar. As the velocity increased, a point was reached at which the dye stream became wavy and then diffused throughout the tube. The flow had become turbulent. By careful manipulation, Reynolds was able to obtain a value of Re = 12,000 before turbulence occurred. However, this number has significance only for the specific flow geometry under consideration. In general the roughness of the fluid boundaries (e.g., rocks in the stream channel) will influence the onset of turbulence, as will the flow geometry. For a wide stream, turbulent flow occurs for $Re \geq 500$ where the mean velocity and stream depth are used for v and ℓ (Eagleson 1970). For standard temperature and pressure, this turbulence criterion becomes $v\ell \geq 6 \times 10^{-4}\ m^2\ sec^{-1}$. Thus, except for extremely shallow streams and/or low velocities, turbulent flow is the rule.

The nature of the flow is characterized by the value of its Reynolds number. The Reynolds number may be considered as the ratio of turbulent tendencies to stabilizing tendencies or, as stated commonly in physics texts, inertial forces (producing changes in velocity) to viscous forces (tending to oppose velocity changes). Thus, for large values of Re the inertial forces dominate and the flow is turbulent; for small values of Re the stabilizing or viscous forces dominate and the flow is laminar.



Reynolds' Apparatus

Steady and Unsteady Flow

Before we get into a discussion of steady and unsteady flow, we need to realize that the conditions at a point in the flow are functions of many variables. For example, the density may be a function of the temperature, time and position, where density is the dependent variable and temperature, time and position are the independent variables. For our discussion, velocity will be the important dependent variable and time and position the most important independent variables.

In the stream, the amount of water flowing through a section of the stream may vary with time depending on conditions upstream such as rainfall and snowmelt, or it may remain constant with time. This time dependence is what distinguishes steady and unsteady flow. For steady flow the conditions at any point in the flow remain constant over time. Thus,

$$\frac{\partial v}{\partial t} = 0.$$

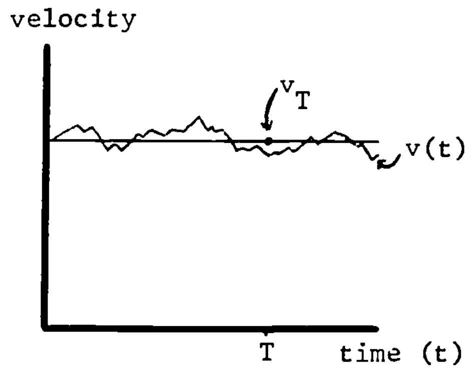
Likewise, if the velocity is a function of time and the amount of water flowing through that section of the stream is variable then the flow is unsteady. Thus,

$$\frac{\partial v}{\partial t} \neq 0.$$

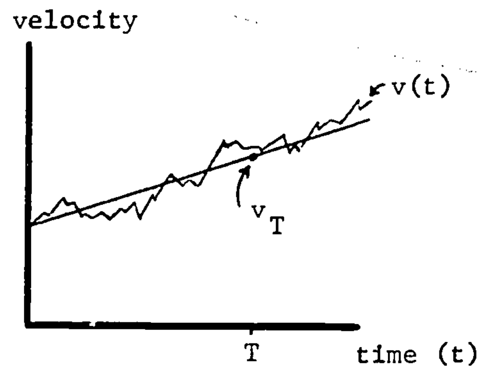
Since in turbulent flow small fluctuations always occur at any point, it would seem that turbulent flow would always be unsteady. This is not strictly true, however. If the velocity pattern is as illustrated below, then we can say that the mean velocity is steady. Where the term

$$v_T = \frac{1}{T} \int_0^T v \, dt$$

called the turbulent mean velocity, does not change with time, we define the turbulent flow as steady. The time T is the turbulent time scale of interest (*see* e.g., Tennekes and Lumley 1972, Simpson 1979).



Steady

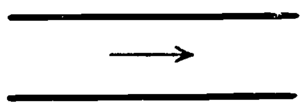


Unsteady

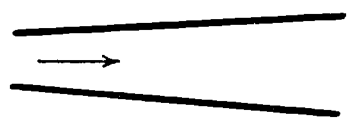
As mentioned previously, sometimes the equations describing a particular flow situation can be very complicated. Derivatives in time as well as in space occur commonly in such situations. If we can describe the flow as being "steady," the time derivatives are not included in the equations and the solutions thus found are time invariant. These are referred to as steady state solutions. These steady state solutions can usually be found more easily. At the minimum, the independence on time removes one more complication in finding a solution.

Uniform and Nonuniform Flow

For steady flow the velocity at a point is constant with time. For uniform flow, we have a similar criterion; in this case, the velocity is the same in some direction at a given instant. Thus $\frac{\partial v}{\partial s} = 0$, where s is a directional coordinate. This condition is satisfied in the case of a stream if it has identical cross sections throughout the region of interest. If, however, the cross sections vary, then the velocity would be a function of s and $\frac{\partial v}{\partial s} \neq 0$ and the flow would be classified as nonuniform. This will become much clearer as the concept of continuity is mastered in subsequent sections. For example, flow of a stream through a long uniform section is uniform. Similarly, water flowing through a narrowing section or a curved section of the stream is nonuniform.



Uniform



Nonuniform

Compressible and Incompressible Flow

The density of the fluid is an important characteristic which enters into many flow equations. In considering the flow of gases especially, the density of the gas can change in the flow considered and cannot always be treated as a constant in the solution of the problem. The density of a gas also changes readily with temperature and pressure. Liquids such as water are generally considered incompressible. The density does change with temperature and pressure, but the changes are small and thus treating the liquid density as constant is acceptable.

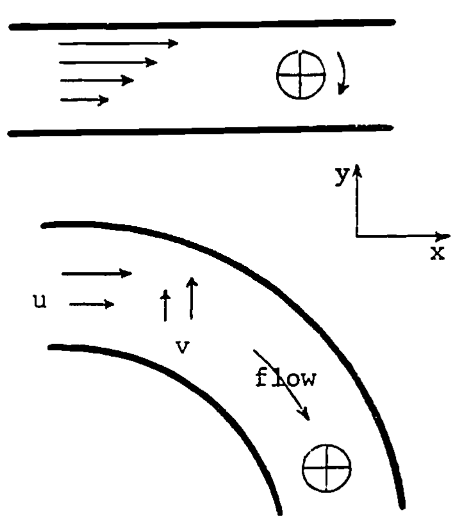
Ideal Fluid Flow

At one time or other we have all placed our hand in a flowing stream, and noticed a force exerted in the direction of flow. This force is known as drag, and it is due to the action of frictionally induced stress in the fluid layers near the object (*see* Vennard and Street 1975, Chapter 13, for further discussion of drag). As pointed out in an earlier module of this series, the frictional stress is due to the fluid viscosity, which is in turn caused by cohesive forces between molecules. It turns out, however, that these frictional forces are significant only in the vicinity of the object. If we consider large expanses of fluid, such as a large river or the center section of a stream away from the frictional effects of streambed or banks, we can safely neglect the frictional effects for this region of flow. In effect, then, we are saying that the fluid is devoid of viscosity in these areas, or inviscid. Such a fluid is termed an ideal fluid.

The concept of ideal fluid flow has important applications. Division of the flow into two regions is possible. For example, away from the edges of flow, or away from flow obstructions such as bridge piers or pilings, it is a good assumption in many cases to treat open channel flow of water as ideal flow. This greatly simplifies the relevant equations while preserving an adequate level of accuracy. More complicated analysis is required in the other flow region, termed the boundary layer, where frictional effects are important. Usually, however, this second region is small compared to the region of ideal flow.

Rotational and Irrotational Flow

Up to this point, we have been able to describe the general flow characteristics of the various sections of the stream. The concepts of rotational and irrotational flow allow a further step toward mathematical simplification when dealing with ideal fluid flow. If an ideal fluid flow is also irrotational then it is defined as potential flow. Although rotation implies a circular motion, motion along a curved path is not necessarily rotational. For example, the common whirlpool or tornado is irrotational. Strictly speaking, under the assumptions of irrotationality as applied to flow situations, this concept would be best understood by considering a small wheel placed in the stream. If the wheel rotates, then the flow is rotational, if not the flow is irrotational, no matter whether the stream follows curved or straight courses. This effect can be convincingly demonstrated with the "drainhole vortex" which frequently occurs when a tank is drained through an orifice in the bottom. This flow closely approximates an irrotational vortex.



Rotational

$$\frac{\partial v}{\partial x} = 0 \text{ but } \frac{\partial u}{\partial y} \neq 0$$

Irrotational

$$\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$$

Mathematically, rotation is defined as:

$$w = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right] \quad ,$$

where u and v are the velocities in the x and y directions, respectively. Therefore, if $w \neq 0$ the flow is rotational and if $w = 0$ or, likewise, $\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}$, then the flow is irrotational.

Rotation commonly arises through the effects of viscosity. Since ideal fluids are assumed to have zero viscosity, they can be considered irrotational (except perhaps for isolated points). As mentioned previously, no real fluid has zero viscosity and thus potential flow never truly exists; however, in many situations such as the large expanse of fluid in the center of a stream, frictional effects are so unimportant that for description of flow in ecosystems, potential flow can be assumed. Analysis of flow situations by assuming potential flow conditions can be a very powerful tool because of the simplified equations of motion. Commonly, the potential flow problem is solved for the "outer" layer of flow. This solution gives approximate values of velocity and pressure on the edge of the boundary layer, allowing solution for the "inner" layer where viscous effects are important. Thus, potential flow theory is a much more powerful tool than might be first assumed.

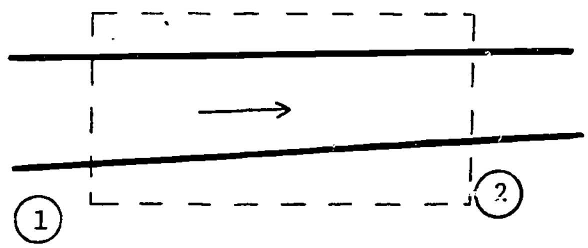
This point marks the conclusion of the first part of this module, in which the basic flow classifications and definitions of elementary fluid dynamics were discussed qualitatively. Based on this discussion the reader should be able to consider a wide variety of naturally occurring flows and be able to classify them, at least in a general sense. With this background, we are now ready to delve into specific cases of interest. As indicated earlier, the remainder of this module will be concerned with conservation of mass in the form of the continuity equation. Treatment of the momentum equations, also known as the equations of motion, is deferred for the present time.

CONTINUITY

Not only are we interested in being able to describe the types of flows that we must deal with, but usually we are interested in the velocity and discharge (amount of water flowing through a section of a stream in a given time) of particular streams. These characteristics may find application in comparison of streams and stream classification (*see* the section "Stream Order and Stream Classification"). They are also important in terms of flood control, control of sedimentation and gauging of available water for downstream use (*see* p. 22). One gross method of determining discharge and velocity for a section of a stream is by using the concept of continuity. For other methods, *see*, e.g., Vennard and Street (1975).

One-Dimensional (1-D) Incompressible, Steady Flow

The concept of continuity can be developed from the general principle of conservation of mass. We shall consider a stream section as illustrated below,



and require that the flow be steady, so that there are no variations in water surface level. Then, mass conservation requires simply that the mass (m) within a given section of flow remain constant with time, i.e., $\frac{dm}{dt} = 0$. Therefore, the amount of fluid entering the section per unit time must equal the amount leaving in the same unit of time. Stated simply:

$$\left(\frac{\text{Mass}}{\text{Time}}\right)_{\text{In}} = \left(\frac{\text{Mass}}{\text{Time}}\right)_{\text{Out}}.$$

Mass per unit time at any point is equal to the product ρvA where ρ is the mass density of the fluid, v is the average velocity, and A is the cross-sectional area at that point. For the section of river shown above in steady flow this becomes:

$$\rho_1 v_1 A_1 = \rho_2 v_2 A_2. \tag{2}$$

For the sake of simplicity, we have assumed a uniform velocity distribution across the stream section. In practice, variation in stream velocity with both depth and width is taken into account by empirically determining average velocity for many sections of the stream and then adding the results. If the fluid is incompressible, then $\rho_1 = \rho_2$ and (2) reduces to:

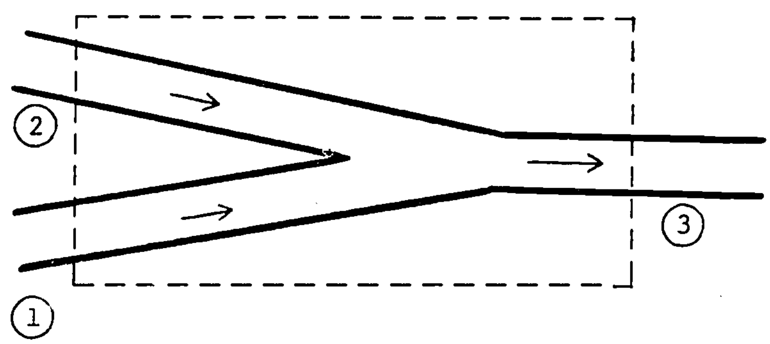
$$v_1A_1 = v_2A_2 \tag{3}$$

This is the very simple one-dimensional form of the continuity equation.

This simple expression for the 1-D continuity equation can be extended to represent cases of more than one input or output to a section of a river. For example, the continuity equation at the confluence of two channels can be represented simply by:

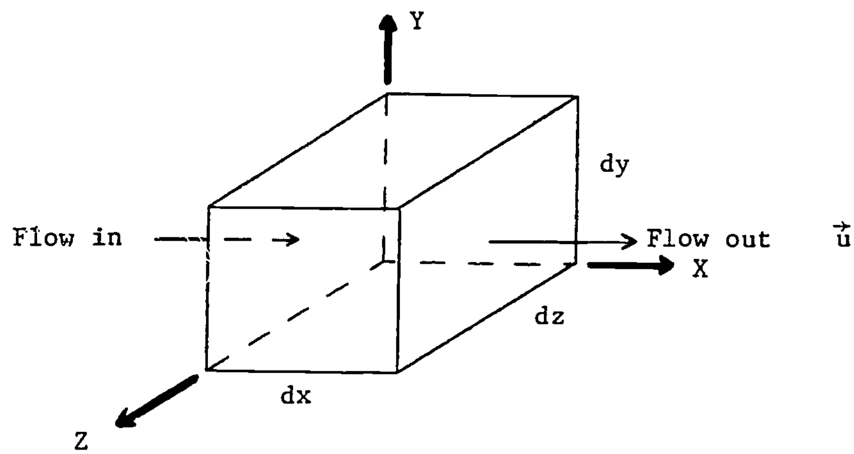
$$Q_1 + Q_2 = Q_3$$

for steady-incompressible flow. From this expression a knowledge of any two of the variables permits a solution for the third.



The General Continuity Equation

In both of the examples given above the flow has been assumed to be 1-D, incompressible and steady. Not all flows are accurately represented by this rather simple case. For example, we may decide that the flow is unsteady or that two or three dimensions are really needed to describe the flow accurately. Thus, we must really look for more general forms of the continuity equation which can apply to these more complex cases.



Consider the mass flux through a small, constant volume element of the stream. The volume may be represented as a cube of sides dx, dy, dz. ρ is the density at any position and time, and $\vec{u}$ is the velocity vector with components u, v, w in the x, y, z directions, respectively. The total mass in the volume dxdydz is ρdxdydz. Hence, the rate of change of total mass with time is $\frac{\partial}{\partial t}(\rho dxdydz)$. Applying the results of the previous section, the total flow through the area dydz at x = 0 is ρudydz. Since we can make our volume element arbitrarily small, we can assume that the flow rate changes linearly with distance. Hence the change in flow from x = 0 to x = dx is equal to the product of the change in flow as a function of x times the distance dx, which can be written:

$$\frac{\partial(\rho u)}{\partial x}\, dx. \qquad (4)$$

The total flow through the area dydz at x = dx is equal to the flow through the area dydz at x = 0 plus the change in the flow between x = 0 and x = dx, or

$$\left[\frac{\partial(\rho u)}{\partial x} dx + \rho u\right] dydz. \tag{5}$$

The difference between the flows through these two faces represents the net accumulation of mass in the volume due to flow in the x direction, and is written (subtracting 4 from 5):

$$\text{Flow in} - \text{Flow out} = -\frac{\partial(\rho u)}{\partial x} dxdydz.$$

If we now repeat this process for the y and z directions, and then equate the rate of change of total mass with the net accumulation, we have:

$$\frac{\partial}{\partial t}(\rho dxdydz) = \left[-\frac{\partial(\rho u)}{\partial x} - \frac{\partial(\rho v)}{\partial y} - \frac{\partial(\rho w)}{\partial z}\right] dxdydz.$$

Dividing through by dxdydz, we get the general equation of continuity for compressible or incompressible fluids:

$$\frac{\partial \rho}{\partial t} = -\frac{\partial(\rho u)}{\partial x} - \frac{\partial(\rho v)}{\partial y} - \frac{\partial(\rho w)}{\partial z} = \vec{\nabla} \cdot (\rho\vec{u}), \tag{6}$$

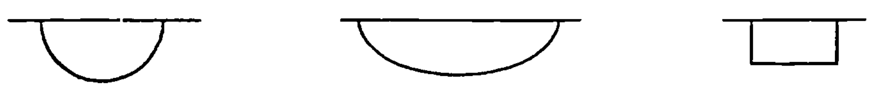
where we have written the right-hand side in conventional vector notation as the divergence of the vector quantity $\rho\vec{u}$. For the case of incompressible flow, we have $\frac{\partial \rho}{\partial t} = 0$, so that we get the very useful expression:

$$\vec{\nabla} \cdot \vec{u} = 0.$$

In other words, the net volume efflux at a point must be zero for incompressible flow (assuming no sources or sinks). Note that the use of the divergence symbol emphasizes that the continuity relationship is a general expression holding for arbitrary coordinate systems.

Coordinate Systems

Before trying to solve a complex mathematical description of a particular flow, it is wise to consider which coordinate system is most appropriate for the problem. Most people are familiar with the Cartesian coordinate system used above to develop the continuity relationship. Other coordinate systems do exist, however. For example, the polar coordinate system may reduce the complexity of a problem which has circular symmetries, or circular boundaries. Likewise, spherical and elliptical coordinate systems are appropriate when problems contain spherical or elliptical symmetries, respectively. Examine the examples of stream cross sections below:



A. B. C.

Although we may be dealing with the same general mathematical problem, it turns out that the problem solution can be considerably simplified in many cases by choosing a coordinate system whose symmetries match those of the particular problem at hand. Polar, elliptical, and Cartesian coordinate systems should probably be chosen for A, B, and C, respectively.

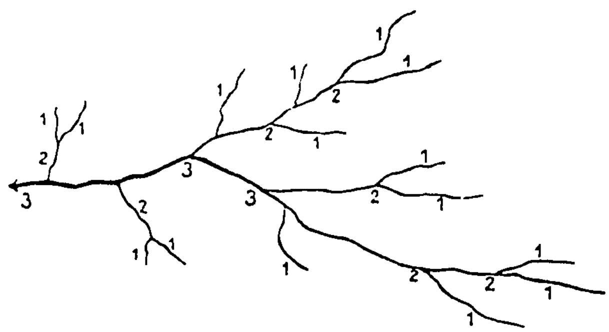
Most of the general equations for motion and continuity are therefore given in vector notation to suggest that we consider the appropriate coordinate system for the problem. Expansions of the vector symbols for coordinate systems and examples of coordinate systems may be found in many fluid mechanics texts as well as general mathematical or engineering reference books.

Average Velocity

Throughout the previous development of the continuity equation, we have referred to the average velocity. In a real fluid the velocity will vary across the cross section because of the action of viscosity. For example, the fluid at the boundary has zero velocity due to a "no-slip" criterion, so that in the vertical direction the velocity profile is not represented by a straight line equal to the average velocity. Instead the velocity profile can be represented by a parabola (*see* the diagram in the section "Applications to Hydrology"). The fact that the velocity varies is really important to the organisms living in aquatic situations. Small animals living in the boundary layer can avoid the strong velocities and the strong forces associated with them. Others living in the main flow must learn to deal with them. Therefore, the average velocity may be used to compare different streams or rivers but the real velocity that an organism encounters may be quite different.

STREAM ORDER AND STREAM CLASSIFICATION

In the first decades of this century, classification of lakes was introduced as a means by which different lakes of similar types could be compared. Until recently, no generally acceptable means of classifying running water was available. Former classification schemes were based on productivity, chemical content, indicator species, and substrate. Recently a method developed by geologists involving stream order has attained widespread use and acceptance. Termed drainage analysis, this technique is based on a system of classification in which tiny, fingertip elements of a stream system are called first order units, and higher orders are created downstream whenever streams of equal order join, as shown below. The diagram is from Abell (1961).



Note that low order streams do not affect the classification of higher orders upon entry. Photogrammetric maps on a scale of 1:30000 or less can be used almost directly for the drainage analysis. In order to eliminate strictly ephemeral streams, the biologist must add the concept of "biological significance." This criterion could be based on consistency of species occurrence. This is impossible to accomplish from map-based drainage analysis and must rely instead on extensive and repeated field work.

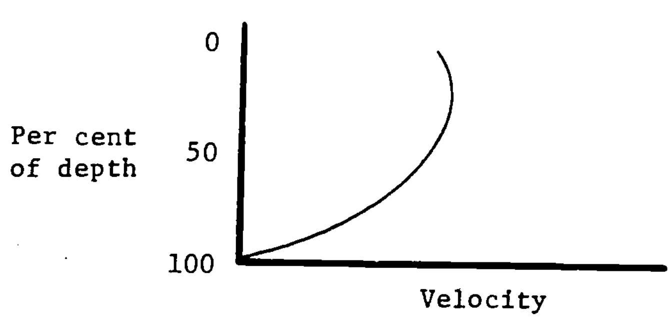
Stream order has been found to be related empirically to discharge, stream length and other characteristics of the stream. These relationships are not exact and care should be taken to avoid obvious discrepancies such as sections that are broader or deeper than predicted from stream order. On the whole, streams do fit well into the classification scheme. As mentioned previously, the discharge can be related to mean velocity and thus the discussion of that section is relevant. Also, velocity influences the type and size of subtrate. Different types of substrate influence the types of organisms that can live there. Likewise, stream length is a rough indication of the amount of habitat available for colonization. Therefore, a stream order can be related to the types and numbers of organisms in a stream section (Harrel 1967). Although this discussion is rather crude, the concept of stream classification and comparison is based on arguments such as these. It should be pointed out that not all limnologists agree that classification of this type is useful or correct or even that streams can or should be classified. For further information, *see* Abell (1961), Illies and Botosaneanu (1963), and Kuehne (1962).

APPLICATIONS TO HYDROLOGY

Hydrology is one area in which the continuity relationship has direct application. Here we are interested in being able to predict discharges from streams so as to determine the amounts of water available for use or to predict floods or droughts.

As we can see from the previous discussion, in order to measure the discharge of a river or a stream, we need to be able to measure the cross-sectional area and the average velocity. The cross-sectional area can easily be measured by sounding with a line and weight or by rods. The cross-sectional area is dependent on the contour of the bed. The rougher the bed, the more measurements that will have to be made. Care must be taken to ensure that the line and rod are vertical so that errors will be minimized (Hoyt 1907). The velocity may be measured by observing the time of passage of a float over a measured course, or by noting the revolutions of the wheel of a current meter or by measuring the slope of the stream and using empirical slope formulas. Discharge measurements are usually classified in accordance with the method used to determine the velocity (Hoyt 1907). In determining the discharge, we are not interested in just any velocity in the cross section but we wish to know the mean velocity.

The velocity of flowing water depends principally on (1) the slope of the stream, (2) roughness of the streambed and (3) the hydraulic radius (Eagleson 1970, Hoyt 1907, Johnstone and Cross 1949, Kazman 1965, Mead 1950, Meyer 1928). The slope of the stream (S) in turn depends on the slope of the streambed and on other channel conditions. The hydraulic radius (R) is the area of the cross section divided by the wetted perimeter. The roughness of the streambed (n) varies from stream to stream. Values may be found in U.S. Geological Survey material. The mean velocity at a cross section could be determined from the vertical velocity curve. However, this entails performing many measurements. It has been observed that on many streams under various conditions of depth, velocity and roughness of the streambed, the vertical velocity curves are approximately parabolic, as shown below.



The maximum velocity occurs at a point below the surface with the velocity decreasing as it approaches the surface or the streambed. The maximum usually occurs at about one-third of the depth and the mean is very close to .6 of the depth (Hoyt 1907, Johnstone and Cross 1949, Mead 1950). If the stream is very deep, both of these points may lie at a greater depth and if the stream is shallow and has a rough bed, both of these points may be totally erroneous, but the above-mentioned measurements hold approximately for a wide variety of streams.

Thus, one may simply proceed to measure the mean velocity by using the .6 depth measurement. The error is generally small. If, however, greater accuracy is desired, three points should be measured. Measurements at .2, .6 and .8 can be combined to obtain an even better estimate of the mean velocity in the following manner:

$$\frac{v_{.2} + v_{.8} + 2v_{.6}}{4} = v_m \ .$$

It is common practice to compute partial discharges from partial cross sections and to combine these to obtain the total discharge (Hoyt 1907). This method allows for a more accurate calculation of the discharge by not assuming that a single velocity measurement can be applied to the whole cross section of the channel.

The float method for measurement is performed by computing the velocity of the flow from measurement of the passage time for a given length of stream (Hoyt 1907). If the conditions of the channel change during the length of the stream used, this can introduce excessive error into the measurement. Also, the surface velocity is not a very good estimate of the mean velocity even when multiplied by a coefficient that will reduce it to approximately the correct value.

Slope measurements combine the effects of roughness and the hydraulic radius in computation of the velocity. One must determine: 1) the mean area of cross section; 2) slope of the surface of the stream; and 3) data on the roughness of the bed from which to determine the value of "n". The velocity can then be determined from any one of a variety of "slope" formulas such as the one by Kutter where S is the slope, n is the roughness and R is the hydraulic radius (Hoyt 1907).

$$v_m = \left[\frac{\frac{1.811}{n} + 41.6 + \frac{.00281}{S}}{1 + \left(41.6 + \frac{.00281}{S}\right)\frac{n}{\sqrt{R}}}\right]\sqrt{RS}$$

The slope method can be used for measuring flood discharge but it is most useful in designing channels to carry a certain discharge at a given velocity (Mead 1950).

Thus, the simple concept of continuity has application in the field of hydrology for determining the discharge of rivers and streams. The applications of the continuity equation may involve measurement of the discharge and cross-sectional area. From this data the average velocity can be determined.

CONCLUSION

Throughout this module we have discussed concepts that are basic to fluid flow and streams. Many of these concepts, especially those dealing with flow classification, will gain greater significance in later modules. However, the basic continuity relationship is readily applicable to hydrology and can form a basis for understanding the relationships of aquatic organisms to running water.

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PROBLEMS

1. You are studying a community of organisms in a stream. In order to be able to understand more exactly the physical and chemical conditions affecting these organisms you decide to formulate a mathematical model of the streamflow. What descriptive terms would you like to be able to use to make the mathematical formulation more tractable, i.e., laminar versus turbulent, etc.? Are these generally realistic?

2. If the flow were considered to be unsteady, how might this affect the composition and distribution of the organisms? What special problems might arise? Would there be similar problems in nonuniform flow? Consider these terms individually.

3. You are out in the field and have decided to study a network of streams. The streams are fairly uniform and so you decide that the average velocity of the sections of the streams may be a reasonable first method for comparing the physical environment of the stream life. How would you go about finding the average velocity of each section? What other method might be used for comparing the streams?

4. In H.B.N. Hynes book, The Ecology of Running Water,* there is a discussion on methods for measuring the discharge of a stream section. One method employed in the field is to apply the following formula:

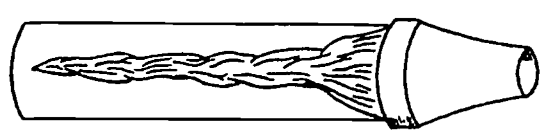
$$Q = \frac{wda\ell}{t}$$

*Hynes, H.B.N. 1970. The ecology of running water. Univ. Toronto Press, Ontario, Canada. 555 pp.

where Q = discharge, w = width, d = mean depth, and ℓ = distance over which a float travels in time t. The term a is a correction coefficient that varies from .8 to .9 depending on the nature of the streambed. Quite frequently an orange is used as a float, because it is conspicuous and floats almost entirely submerged.

How can this method be related to common hydrological methods of measuring discharge? How might a value of "a" be predicted? Why the dependence of "a" on the nature of the streambed?

5. The average velocity measurements calculated from discharge do not give us a very good idea of what the actual velocity is that an organism experiences. A small, simple apparatus was designed by Gessner. Although crude it can give reasonable results. The apparatus consists of a conal end piece with a small opening (small enough to cover by a finger) that is directed into the flow. Inside is a collapsed bag protected by an open-ended cylinder as pictured below:

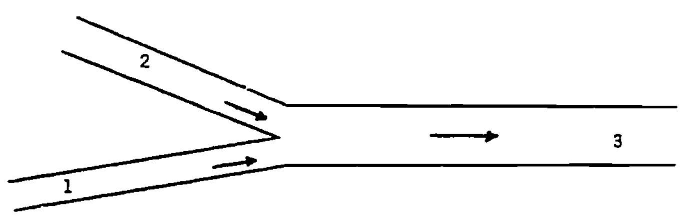


How might you use this apparatus to measure the velocity? Include a description of the physical use and calculation formulas.

6. The previous problems have explored methods for computing the discharge and velocity of a section of a stream. If you are studying a network of streams, how might you check your calculations? If there are discrepancies does this necessarily mean that your measurements are wrong? Where might errors come from?

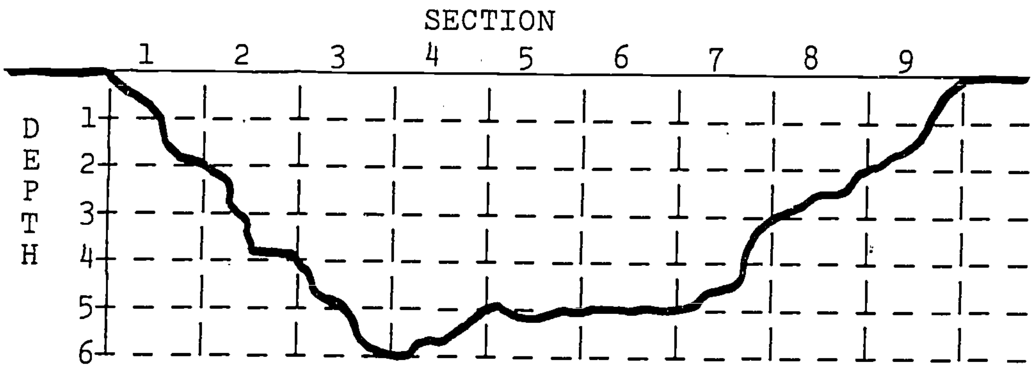
7. The following measurements were taken from a small section of streams. Compute the missing information.

	Width (ft)	Average depth (ft)	Cross-sectional area (ft^3)	$v_{average}$	Q.
1	1	1/2			
2		1	2	1.5 ft/sec	
3		1	3		5 ft^3/sec



What assumptions must you make to perform these calculations?

8. In the hydrology section, we discussed the calculation of discharge for a larg river by adding up partial discharges. Use that method to compute the total discharge for the river cross section given below.



Width = 10 feet for each section

Velocity measurements:

Section 1: v_{avg} = 1 ft/sec

Section 2: v_{avg} = 1.1 ft/sec

Section 3: $v_{.2}$ = 1 $v_{.8}$ = 1 $v_{.6}$ = 1.3

Section 4: $v_{.2}$ = 1 $v_{.8}$ = 1 $v_{.6}$ = 1.4

Section 5: $v_{.2}$ = 1 $v_{.8}$ = 1 $v_{.6}$ = 1.6

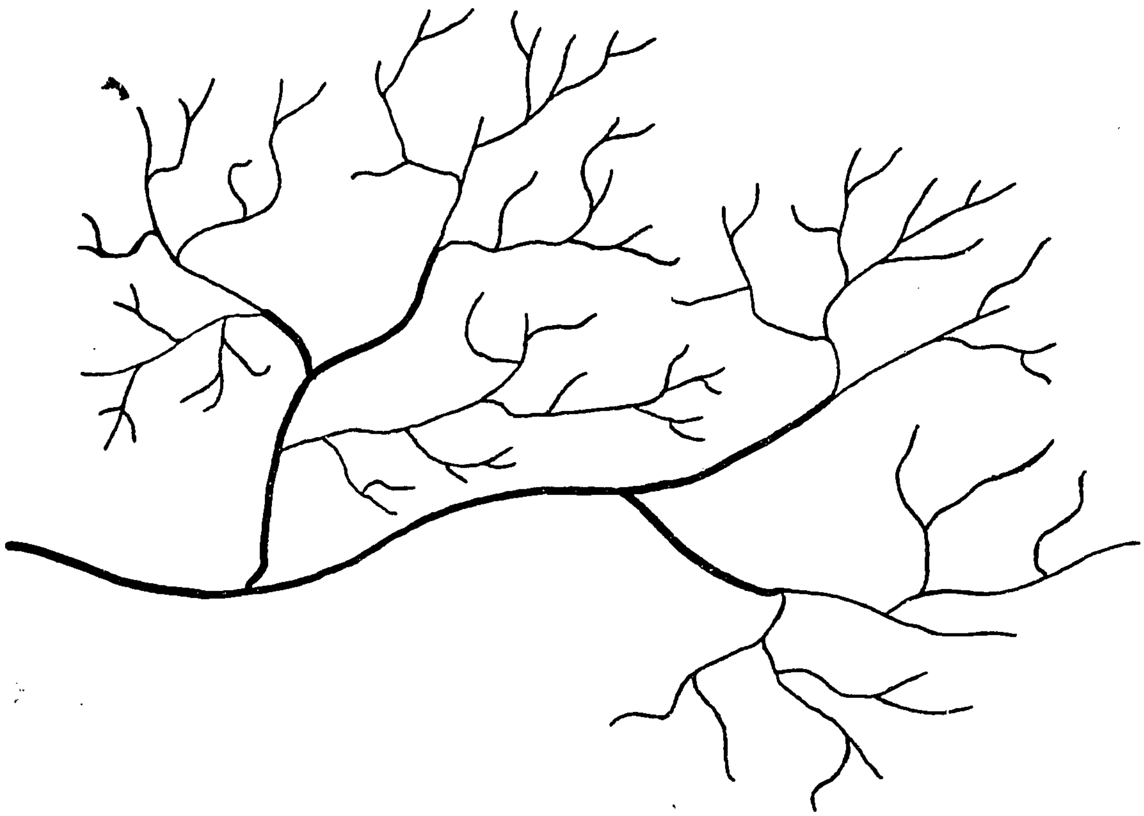
Section 6: $v_{.2}$ = 1.1 $v_{.8}$ = 1 $v_{.6}$ = 1.5

Section 7: $v_{.2}$ = 1 $v_{.8}$ = 1 $v_{.6}$ = 1.2

Section 8: v_{avg} = 1.1

Section 9: v_{avg} = 1

9. A stream network is given below. Determine the order of each of the streams.



PROBLEM SOLUTIONS

1. Generally speaking, we would like to be able to deal with a steady state problem, thus eliminating time derivatives from our model. Steady flow also allows one to deal with time averaged mean quantities, allowing the more straightforward analysis developed for laminar flow to be applied. Steady state conditions are realizable on most streams over time spans short with respect to a year, assuming no events such as a recent rainfall have temporarily caused unsteady flow. The 1-year limit rules out annual streamflow variations due to seasonal rainfall and/or snowmelt.

   Uniform flow is approximated well in many streams where stream width and depth remain relatively constant. As might be expected, further simplification to the governing equations for our model obtains from this assumption.

   If in our analysis we are interested in the flow away from the streambed or obstructions, the assumption of ideal fluid flow simplifies the relevant equations, as well as making available a large body of already solved problems.

   In passing, it should be recognized that viewing the fluid as incompressible, generally a good assumption for water, is another standard simplifying assumption.

2. Unsteady flow implies that the volume of water flowing through the stream changes with time. This means that the banks of the stream

may change through erosion and deposition of material. As a consequence, organisms that live on the bank of the stream probably must be ephemeral or be able to withstand drought or flood conditions. Also, organisms in the middle of the stream may not set up permanent communities because the streambed may be scoured during floods. More minor changes can cause changes in velocity patterns that would destroy organisms attached to rocks. This is why ephemeral streams have such sparse communities associated with them. Nonuniform flow, however, would not imply impermanence of existing conditions so that organisms could be expected to establish themselves in areas suitable to them. Because the velocity patterns change along the stream, however, one might expect more diversity of organisms than in a uniform stream.

3. In this case any of the methods in the section "Applications to Hydrology" could be applied to measure the velocity. In a small stream it would probably be more realistic to approximate the cross-sectional area and measure the discharge by placing a weir in the stream. The velocity could then be determined from the measurements via the continuity relationship. The stream could also be analyzed using stream order analysis.

4. Commonly discharge is defined as

$$Q = Av$$

$$wd = A, \quad a\ \frac{\ell}{t} = v$$

Therefore the two methods are comparable. "a" could probably be predicted from measurements of the roughness of the streambed as well as from an idea of the relationship between the surface velocity and the mean velocity. Exact determination of "a" is difficult. The value of "a" is dependent on the nature of the streambed because the streambed determines, in part, the form of the velocity curve. A rough or smooth streambed could shift the position of the mean velocity as well as its value.

5. This apparatus can be used by placing one's finger over the opening and placing the apparatus in the vicinity of which you wish to determine the velocity. The finger is removed for a certain length of time and the amount of water collected in the bag per unit time is measured. With a knowledge of the cross-sectional area of the opening, the velocity can be determined by applying the continuity relationship

$$v = \frac{\text{amount of water}}{\text{time interval}} \times \frac{1}{\text{cross-sectional area}}$$

6. If one was studying a network of streams, consistence in the measurements can be checked by using the continuity relationship. For example, if there were two streams joining to form a third then $Q_1 + Q_2 = Q_3$. If this continuity relation is not satisfied, the error is probably in the measurements, although not necessarily so. For example, water may be lost or gained through deep seepage or underground flow. While difficult, if not impossible, to measure, generally such effects are small. A more probable cause of the discrepancy would be due simply to the uncertainty of the measurements

in a statistical sense. To allow for this, one should take several measurements of each point to establish some sort of confidence in the easurements in a statistical sense.

7.

	Width (ft)	Average depth (ft)	Cross-sectional ft^2	$v_{average}$	Q
1	1	1/2	1/2	4	2.0
2	2	1	2	1.5	3.0
3	3	1	3	1.67	5

You must assume that there are no other inflows or outflows.

8. Section 1: 1 ft/sec x 1 ft x 10 ft = 10 ft^3/sec

Section 2: 1.1 ft/sec x 10 ft x 3 ft = 33 ft^3/sec

Section 3: $v_{avg} = \frac{1 + 1 + 2(1.3)}{4} = 1.15$ ft/sec

1.15 ft/sec x 5 ft x 10 ft = 57.5 ft^3/sec

Section 4: $\frac{1.1 + 1 + 2(1.4)}{4} = 1.20$

1.225 x 10 ft x 5.5 ft = 66.0

Section 5: $\frac{1.1 + 1 + 2(1.6)}{4} = 1.30$

1.325 x 10 ft x 5 ft = 65.0

Section 6: $\frac{1.1 + 1 + 2(1.5)}{4} = 1.275$

1.275 x 10 ft x 5 ft = 63.75

Section 7: $\frac{1 + 1 + 2(1.2)}{4} = 1.1$

1.1 x 10 ft x 4 ft = 44

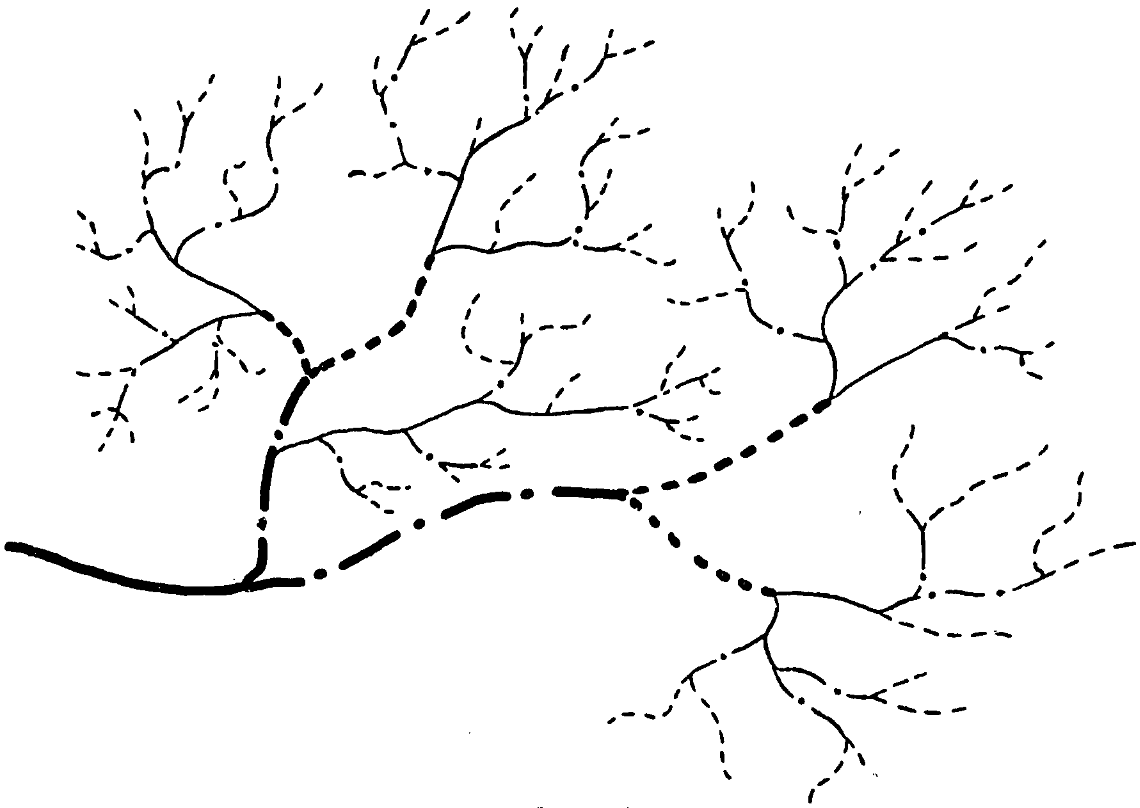
Section 8: 1.1 x 10 ft x 2.5 ft = 27.5

Section 9: 1 x 10 x 1 = 10 ft^3/sec

Q_{Total} = 10 + 33 + 57.5 + 66.0 + 65.0 + 63.75 + 44 + 27.5 + 10

= 376.75 ft^3/sec

9.



Legend

First-order channel
Second-order channel
Third-order channel
Fourth-order channel
Fifth-order channel
Sixth-order channel

APPENDIX. Symbols, Units and Dimensions

Symbol	Quantity	Unit	Dimension
A	area	m^2	L^2
a	correction coefficient	dimensionless	
D	diameter	m	L
d	mean depth	m	L
ℓ	linear dimension	m	L
n	streambed roughness	m	L
Q	discharge	$m^3\ s^{-1}$	L^3T^{-1}
R	hydraulic radius	m	L
Re	Reynolds number	dimensionless	
S	slope of the stream	dimensionless	
s	directional coordinate	dimensionless	
T	turbulent time scale	s	T
t	time	s	T
v	velocity	$m\ s^{-1}$	LT^{-1}
v_m	mean velocity	$m\ s^{-1}$	LT^{-1}
v_T	mean velocity	$m\ s^{-1}$	LT^{-1}
u	velocity	$m\ s^{-1}$	LT^{-1}
w	velocity	$m\ s^{-1}$	LT^{-1}
w	rotation	dimensionless	
w	width	m	L
x	directional coordinate	1 m	L
y	directional coordinate	1 m	L
z	directional coordinate	1 m	L
ρ	mass density	$kg\ m^{-3}$	ML^{-3}
μ	viscosity	$kg\ m^{-1}\ s^{-1}$	$ML^{-1}T^{-1}$